Arithmetic Series:

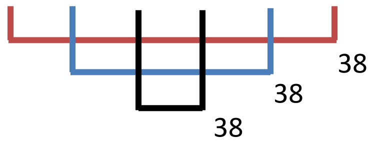
Suppose you wanted to find the partial sum: $S_n = 4 + 10 + 16 + 22 + \cdots + 346$

Method 1: ❶ Figure out all of the missing numbers ❷ Add them all up without making mistakes

Sn = 4+10+16+22+28+34+40+46+52+58+64+70+76+82+88+94+100+106+112+118+124+130+136+142+148+154+160+166+172+178+184+190+196+202+208+214+220+226+232+238+244+250+256+262+268+274+280+286+292+298+304+310+316+322+328+334+340+346 =

Method 2: Use a formula

Let's start with an easier arithmetic partial sum: $S_n = 4 + 10 + 16 + 22 + 28 + 34$



Note the beautiful pattern →

So the sum is: $S_n = 3 \cdot 38 =$ 114

There are six terms, so n = 6

38 = 4 + 34 = $a_1 + a_n$ 3 = $\frac{6}{2} = \frac{n}{2}$

So the formula for the partial sum of an arithmetic series is: $S_n = \frac{n}{2}(a_1 + a_n)$

Ex: Find $S_n = 1 + 5 + 9 + 13 + 17 + 21 + 25 + 29 + 33 + 37 + 41$

$n = 11, \quad a_1 = 1, \quad a_n = 41$ $S_n = \frac{11}{2}(1 + 41) = \frac{11}{2} \cdot 42 = \frac{462}{2} = 231$

Ex: Find

$$S_n = \sum_{n=1}^{145} 3 - 8(n-1)$$

$n = 145 - 1 + 1 = 145, \quad a_1 = 3 - 8(1 - 1) = 3, \quad a_n = 3 - 8(145 - 1) = -1149$

$$S_n = \frac{145}{2}(3 - 1149) = \frac{145}{2}(-1146) = \frac{-166170}{2} = -83085$$

Ex: Find $S_n = 1 + 5 + 9 + \cdots + 805$

$a_1 = 1, \quad a_n = 805, \quad n = ?$ ❯ $805 = 1 + 4(n - 1)$ ❯ $804 = 4(n - 1)$ ❯ $201 = n - 1$ ❯ $n = 202$

$$S_n = \frac{202}{2}(1 + 805) = \frac{202}{2}(806) = 101 \cdot 806 = 81406$$

Ex: Find $S_n = 6 + 7 + 8 + 9 + \cdots$ = this thing never ends. It just keeps getting bigger and bigger.

SO THERE IS NO SUM for ANY infinite arithmetic sequence!!!!! (write NO SUM as your answer)